

Seminar - Selected topics in algebraic and differential topology

Webpage: <https://www.mathematik.hu-berlin.de/~kegemarc/WS2021SeminarAlgTopo.html>

OneNote Link: <https://1drv.ms/u/s!AjhcHi01JrMRgQVPq-GmOPLTveF1>

DE RHAM COHOMOLOGY:

DAVID SVCHODOLL

$$C^k(M) = \{ k\text{-forms on } M \}$$

smooth mfd

$$d: C^k \longrightarrow C^{k+1}$$
$$w \longmapsto dw$$

THM: $d^2 = 0$

$$H_{dR}^k(M) = \text{cohomology of } (C^k \xrightarrow{d} C^{k+1} \xrightarrow{d} C^{k+2} \dots)$$

THM: $H_{dR}^k(M) \cong H^k(M; \mathbb{R})$

* OBSTRUCTION THEORY & HOMOTOPY CHAR OF COHOMOLOGY

$$H^i(X; G) = [X, K(G, i)]$$

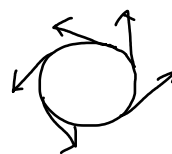
↑
bilinear Mac Lane space

* CHARACTERISTIC CLASS

$\square Q \Rightarrow TM^n$ trivial i.e. $TM \cong M \times \mathbb{R}^n$?

Ex: S^1 YES!

S^2 NO!



$$\nexists TM \text{ trivial} \Rightarrow C(TM) = 0$$

* HOMOLOGY OF FIBRATIONS :

THEO MÜLLER

$F \longrightarrow X \longrightarrow B$ a fiber bundle

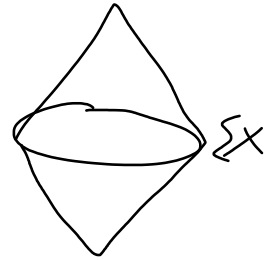
easy to compute $\pi_k(X)$

how to $\parallel H_k(X)$? \longrightarrow spectral sequences

* FREUDENTHAL SUSPENSION THM :

$\Sigma : X \longrightarrow \Sigma X$

$\pi_k(X) \longrightarrow \pi_{k+1}(\Sigma X)$

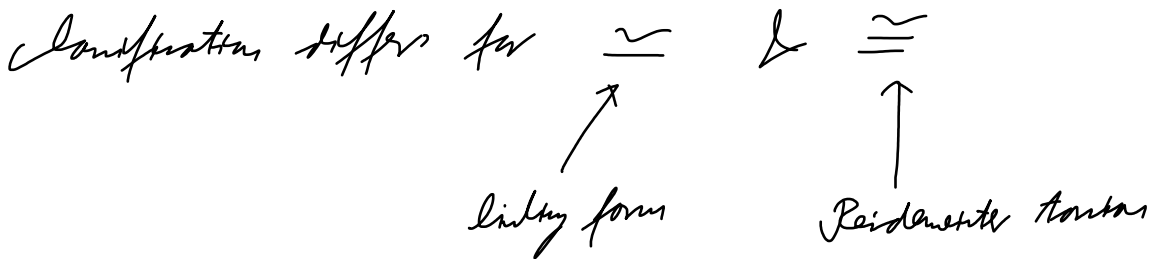


is an iso for $1 \leq k \leq 2n$ if X is n -connected

$X = S^n \longrightarrow$ stable homotopy groups

* Lens spaces :

$L(p, q) = S^3 \times D^2 \cup_{\varphi_{q,1}} S^3 \times D^2 = S^3 / \mathbb{Z}_p$



SMALE'S THM :

$O(3) \xrightarrow{\cong} \text{Diff}(S^2)$ in a hom. equivalence

HATCHER : $O(4) \xrightarrow{\cong} \text{Diff}(S^3)$

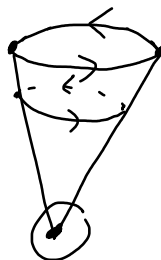
$O(5) \neq \text{Diff}(S^4)$

* \check{C} ECH HOMOLOGY

from. along with open covers of the space

* Orbifolds : M/G i.e., NOT a manifold, but 'often' an orbifold

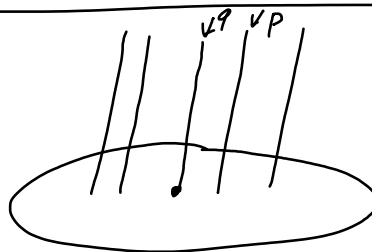
$$D^3 /_{X \sim -X} = \mathbb{C}P^2$$



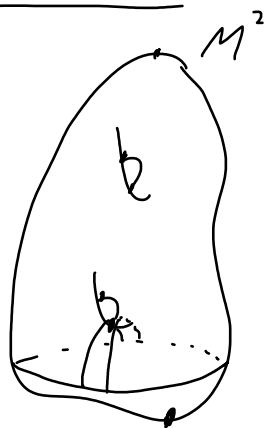
* Seifert fibered spaces :

S^1 -bundle over orbifolds

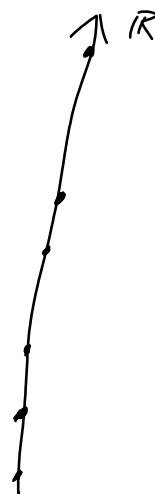
ex: $L(P, q)$



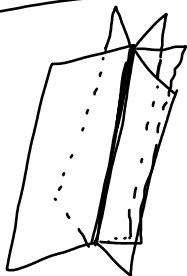
MORSE THEORY : (CHRISTOPH KROPF)



\hookrightarrow Morse



* OPEN BOOKS



$\forall M^3 \exists$ open book

(=) $\forall M^3 \exists$ subset $K \subset M$ s.t.

$M \setminus K$ fibers over S^1

ex: \bigcirc \bigoplus , fig 8

NOT all